CALCULATION OF THE DURATION OF COOLING OF SOLIDS

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UDC 536.21

A simple analytical equation for determining the duration of cooling of solids, especially foodstuffs in the form of plates, is obtained.

Existing analytical methods of determining the duration of cooling of solids (foodstuffs) are complicated for practical use [2-4]. The graphic methods used for this purpose [5-7], give approximate results. In [1] one of the authors carried out analysis of the temperature field during cooling of foodstuffs, and a graphic method was proposed for determining the duration of cooling.

In this article we have given a simple mathematical formula for determining the duration of the cooling process. In addition to this, a functional relationship between the individual parameters which characterize the cooling process is developed graphically.

The temperature field of the plate is described by an infinite series [10]

$$\theta = \sum_{i=1}^{\infty} \frac{2\sin\mu_i}{\mu_i + \sin\mu_i \cos\mu_i} \exp\left(-\mu_i^2 \mathrm{Fo}\right) \cos\left(\mu_i \frac{x}{s}\right), \tag{1}$$

μ_i are the roots of the characteristic equation,

$$\operatorname{ctg}\mu_i = \frac{1}{\mathrm{Bi}}\mu_i.$$

Usually it is important to know the temperature in the middle of the plate, i.e., where $x = 0$. Equation (1) will then assume the form

$$\theta = \sum_{i=1}^{\infty} \frac{2\sin\mu_i}{\mu_i + \sin\mu_i \cos\mu_i} \exp\left(-\mu_i^2 \mathrm{Fo}\right). \tag{2}$$

Hence it follows that

$$\theta = f(\mathrm{Bi},\ \mathrm{Fo}).$$

The θ values for various values of Bi $(0.01 - \infty)$, and Fo (0-400) were calculated according to formula (2) where $i = 1, \ldots, 6$. The results obtained were systematized, and part of them was expressed graphically in Fig. 1.

We will look for the functional dependence of θ on Bi and Fo in the form

$$\mathrm{Fo} = -k \lg\theta + n, \tag{3}$$

where

$$k = f(\mathrm{Bi}).$$

The values k and n were determined at different values of Bi; n has the same value 0.12 for all cases.

A method of a tightened thread was used to determine the coefficients k and n [8].

By examining k as a function of Bi (Fig. 2), we obtain the following analytical expression:

$$k = \frac{c}{\mathrm{Bi}} + d.$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 1, pp. 177-179, January, 1970. Original article submitted February 18, 1969.

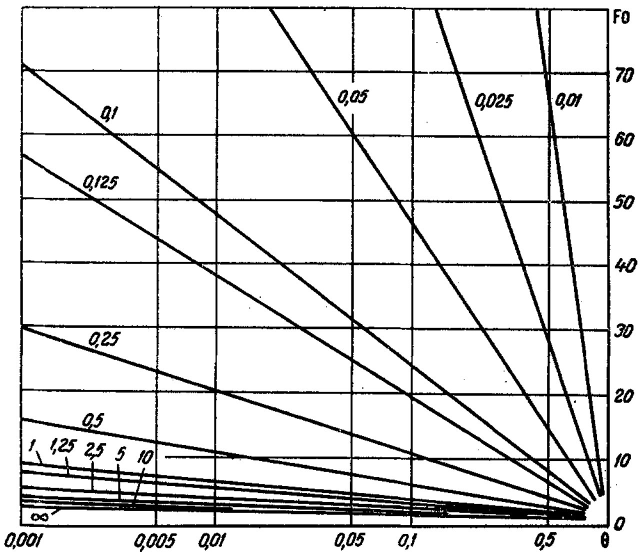

Fig. 1. Relationship between θ and Fo for different values of Bi. The figures on the curves are Bi values.

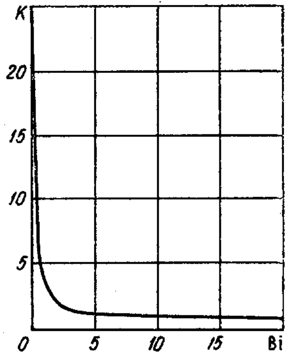

Fig. 2. Relationship between k and Bi.

The coefficients c and d were determined according to a method of mean deviation [8], using a system

$$c\sum_{i=1}^{m} f(x_i) + md = \sum_{i=1}^{m} y_i,$$

$$c\sum_{i=m\pm 1}^{n} f(x_i) + (n-m)\,d = \sum_{i=m\pm 1}^{n} y_i.$$

The number m (which designates the number of observations in the first group) is selected with such a calculation that in the second group it was also equal to m if n was an even number, and equal to m ± 1 if n was an odd number. We found that C = 2.3 and d = 0.8.

Substituting the values obtained for k and n into formula (3), we will find the following analytical expression of the functional relation between F0, Bi, and θ:

$$\mathrm{Fo} = -\left(\frac{2.3}{\mathrm{Bi}} + 0{,}8\right)\lg\theta + 0.12. \qquad (4)$$

Assuming that Fo = $a\tau/s^2$, we obtain

$$\tau = -\frac{s^2}{a}\left[\left(\frac{2.3}{\mathrm{Bi}} + 0{,}8\right)\lg\frac{t_k - t_0}{t_{in} - t_0} - 0.12\right] \qquad (5)$$

If we compare the data which are found by using formulas (4) and (5) with those from series (2), a good agreement of results is noted.

NOTATION

$\vartheta = t_{x\tau} - t_0$;

$t_{x\tau}$ is the temperature at a point at a distance x from the middle plane at a time τ;

t_0 is the temperature of the medium;

$\theta_0 = t_{in} - t_0$;

t_{in} is the initial temperature;

$\theta = \vartheta/\vartheta_0$;

a is the coefficient of diffusivity;

s is half the thickness of the plate;

Fo = $a\tau/s^2$ is the Fourier number;

Bi = $(\alpha/\lambda)s$ is the Biot number.

LITERATURE CITED

1. A. Fikiin, "Variation of the temperature field and determination of the duration of the cooling process in stored products," in: Scientific Works of the Scientific Research Institute of the Starch Industry [in Bulgarian], Vol. 4, Tekhnika, Sofia (1966).
2. D. G. Ryutov, Calculation of the Duration of Cooling of Foodstuffs [in Russian], Scientific Conference of the Moscow Institute of Chemistry, Moscow (1958).

3. Gac, Determination of Cooling of Foodstuffs by Refrigeration, La Revue Générale du Froid, 9, Paris (1964).
4. H. D. Bachr, Kältetechnik, 9, Karlsruhe (1953).
5. H. Gröber and S. Erk, Principles of Heat Transfer [in German], Berlin–Göttingen–Heidelberg, Springer–Verlag (1964).
6. U. Grigull, Temperature Balance in Simple Solids [in German], Berlin–Göttingen–Heidelberg, Springer–Verlag (1964).
7. G. Ivantsov, Heating of Metal [in Russian], Moscow (1948).
8. R. S. Guter and B. V. Ovchinski, Elements for Calculation, Analysis, and Mathematical Treatment of Experimental Data [in Bulgarian], Sofia, Tekhnika (1966).
9. B. Ya. Zel'dovich and A. D. Myshkis, Elements of Applied Mathematics [in Russian], Nauka, Moscow (1967).
10. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Vysshaya Shkola, Moscow (1967).